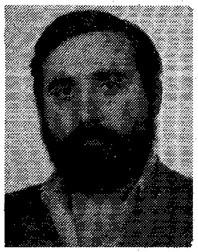

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Matching Network Design Studies for Microwave Transistor Amplifiers

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***Abstract*—Several techniques for design of microwave amplifiers using lumped-element and distributed matching networks are discussed. An extension to the Remez-algorithm design approaches is proposed, whereby gain and ripple quantities may be adjusted using a numerical optimization procedure in order to achieve complete absorption of device model elements and also impedance transformation.**

I. Introduction

IN DESIGNING microwave transistor amplifiers, a widely used technique is that of modeling the active device in terms of simple input and output networks with a gain–slope parameter and subsequently designing matching networks incorporating the model elements and having an inverse gain–frequency slope, e.g., [1]–[4]. Departures from a flat response are caused by a) modeling inaccuracies in output and input admittances, b) the use of a unilateral approximation (see [5]), and c) due to imperfections in the matching network responses. It is usual that computer optimization be used to refine an initial design.

Modeling networks (and gain–bandwidth restrictions which may be derived) are discussed in Section II, with matching network design being treated in Section III. Results are presented in Section IV and conclusions are presented in Section V.

Throughout the discussion, four devices are considered for the purposes of illustration. These are a) the HP 1-μm gate packaged device (FET) [1], b) the Plessey COD package device [2], the HP bipolar device [3], and the HP 1-μm chip device [6]. S-parameter data (50-Ω reference) is given in Table I(a)–(d). Calculation of stability quantities K and B_1 ([7]) indicates that all devices are unconditionally stable within the frequency ranges for which data are available,

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TABLE I
S-PARAMETER DATA FOR TRANSISTORS a)–d)

FREQUENCY	$\|S_{11}\|$	$\|S_{12}\|$	$\|S_{21}\|$	$\|S_{22}\|$	$\angle S_{11}$	$\angle S_{12}$	$\angle S_{21}$	$\angle S_{22}$
4.0 GHz	0.712	0.036	2.589	0.563	-129.0	25.9	74.7	- 42.4
4.5 GHz	0.741	0.038	2.484	0.520	-148.6	20.3	63.1	- 47.9
5.0 GHz	0.682	0.038	2.375	0.485	-161.7	16.7	51.6	- 55.8
5.5 GHz	0.695	0.040	2.229	0.457	-177.2	12.6	38.9	- 64.0
6.0 GHz	0.668	0.043	2.111	0.412	168.1	6.9	26.9	- 74.7
6.5 GHz	0.712	0.045	1.987	0.370	152.4	0.3	15.3	- 83.3
7.0 GHz	0.659	0.047	1.810	0.320	145.2	-5.0	5.3	- 95.9
7.5 GHz	0.695	0.048	1.698	0.266	135.7	-11.0	- 6.4	-109.9
8.0 GHz	0.674	0.050	1.649	0.244	126.7	-16.8	-16.1	-132.0

(a)

FREQUENCY	$\|S_{11}\|$	$\|S_{12}\|$	$\|S_{21}\|$	$\|S_{22}\|$	$\angle S_{11}$	$\angle S_{12}$	$\angle S_{21}$	$\angle S_{22}$
8.0 GHz	0.620	0.066	1.190	0.880	- 64.0	134.0	150.0	-30.0
9.0 GHz	0.460	0.089	1.340	0.840	- 87.0	129.0	140.0	-43.0
10.0 GHz	0.410	0.082	1.220	0.820	- 96.0	138.0	130.0	-38.0
11.0 GHz	0.400	0.082	1.090	0.830	-116.0	138.0	125.0	-45.0
12.0 GHz	0.250	0.087	1.110	0.860	-107.0	154.0	120.0	-42.0

(b)

FREQUENCY	$\|S_{11}\|$	$\|S_{12}\|$	$\|S_{21}\|$	$\|S_{22}\|$	$\angle S_{11}$	$\angle S_{12}$	$\angle S_{21}$	$\angle S_{22}$
1.0 GHz	0.670	0.060	3.200	0.350	160.0	62.0	72.0	- 78.0
1.25 GHz	0.675	0.080	2.700	0.350	154.0	63.0	66.0	- 81.0
1.5 GHz	0.680	0.100	2.200	0.350	147.0	63.0	60.0	- 85.0
1.75 GHz	0.068	0.120	1.880	0.350	140.0	63.0	54.0	- 89.0
2.0 GHz	0.680	0.130	1.570	0.350	134.0	63.0	48.0	- 93.0
2.25 GHz	0.685	0.140	1.490	0.350	128.0	62.0	45.0	- 98.0
2.5 GHz	0.690	0.150	1.260	0.360	122.0	61.0	41.0	-103.0

(c)

FREQUENCY	$\|S_{11}\|$	$\|S_{12}\|$	$\|S_{21}\|$	$\|S_{22}\|$	$\angle S_{11}$	$\angle S_{12}$	$\angle S_{21}$	$\angle S_{22}$
8.0 GHz	0.719	0.038	1.887	0.618	253.8	54.9	99.4	317.4
9.0 GHz	0.712	0.038	1.720	0.623	248.8	56.3	92.0	313.1
10.0 GHz	0.708.	0.038	1.510	0.625	242.2	57.8	85.5	309.3
11.0 GHz	0.705	0.038	1.400	0.627	238.7	59.2	81.7	306.0
12.0 GHz	0.709	0.038	1.310	0.622	235.2	60.6	77.7	304.0

(d)

(a) Transistor a)

(b) Transistor b)

(c) Transistor c)

(d) Transistor d)

with the exception of device b at the lower end of the range.

II. MODELING NETWORKS AND GAIN–BANDWIDTH LIMITATIONS

A. Gain Relations

The circuit to be considered consists of the transistor connected to the terminations by lossless matching networks, as shown in Fig. 1. With *S*-parameters and reflection coefficients normalized to the same reference (conventionally 50 Ω) the transducer power gain is given by [7]

$$G_T = \frac{|S_{21}|^2\left(1-|S_g|^2\right)\left(1-|S_L|^2\right)}{|1-S_gS_{11}-S_LS_{22}+S_gS_L\Delta|^2}, \qquad \Delta = S_{11}S_{22}-S_{12}S_{21}. \tag{1}$$

A convenient approximation for design purposes is to set

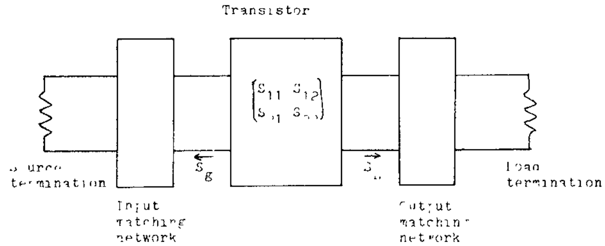


Fig. 1. Amplifier configuration under discussion.

S_{12} equal to 0 with other parameters unchanged. Thus one may write

$$G_u = G_T|_{S_{12}=0} = \underbrace{\frac{|S_{21}|^2}{\left(1-|S_{11}|^2\right)\left(1-|S_{22}|^2\right)}}_{\text{(term 1)}} \cdot \underbrace{\frac{\left(1-|S_{11}|^2\right)\left(1-|S_g|^2\right)}{|1-S_gS_{11}|^2}}_{\text{(term 2)}} \cdot \underbrace{\frac{\left(1-|S_{22}|^2\right)\left(1-|S_L|^2\right)}{|1-S_LS_{22}|^2}}_{\text{(term 3)}} \tag{2}$$

where the first term is $G_{u\max}$, obtained when the ports are conjugately terminated with $S_{12}=0$, and terms 2 and 3 are transducer gains of input and output networks terminated in S_{11} and S_{22}, respectively. The device properties are ordinarily such that $G_{u\max}$ falls with frequency at a rate of about 6 dB per octave and hence to achieve broad-band flat gain it is necessary to design input and output networks so that the product of terms 2 and 3 in (2) gives an inverse gain–frequency slope.

B. Modeling Considerations

It is essential to have simple models for immittance quantities associated with S_{11} and S_{22}, and the elements in these models must be capable of absorption into matching networks. Modeling networks of simple structure using lumped elements and alternatives using commensurate-distributed elements are shown in Fig. 2. (see, e.g., [1], [3]). In the case of commensurate distributed lines, 1/4 wave frequencies of $f_0 = 1.5f_H$ and $f_0 = 2f_H$ were chosen, where f_H is the highest in-band frequency. Element values were calculated for the devices under study using "average," least square, or optimization-based fitting techniques as appropriate. It is noted that the FET chip device could be adequately modeled using *RC* networks, but inclusion of inductive elements was necessary for packaged FET or bipolar devices. A similar statement applies in the case of the commensurate networks using the concept of capacitors and inductors with the transformed frequency variable.

C. Gain–Bandwidth Considerations

Gain–bandwidth studies may be undertaken for the models obtained. These studies are of interest in that they

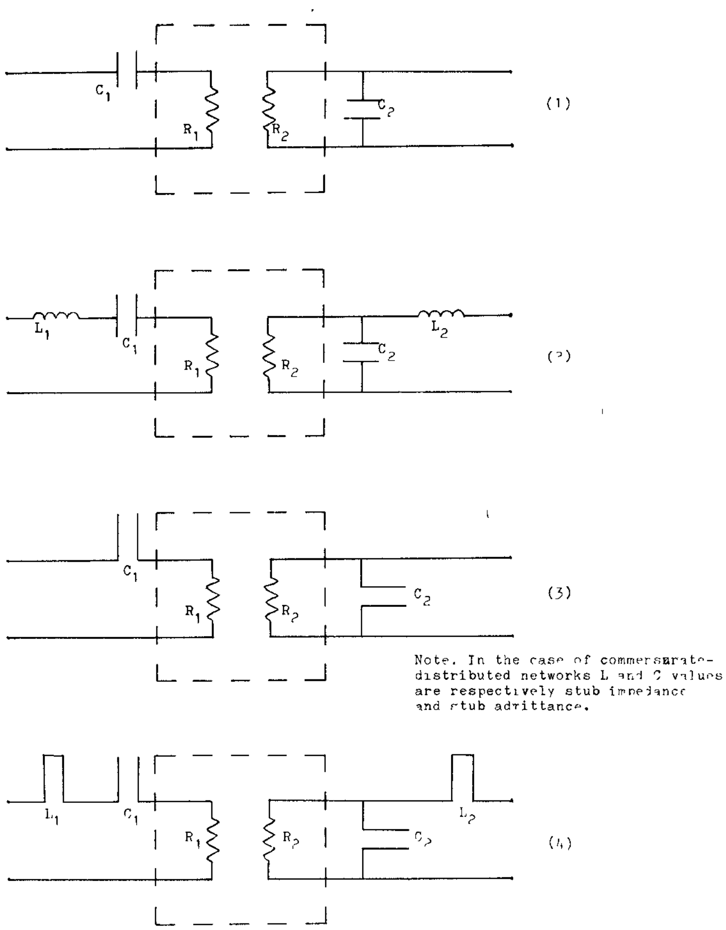


Fig. 2. Lumped and commensurate-distributed networks used to model input and output immitances of devices.

TABLE II
GAIN–BANDWIDTH LIMITATIONS DERIVED FROM ELEMENTS IN MODELING NETWORKS FOR DEVICES STUDIED

Device	Slope $\frac{dB}{oct}$	Band (GHz)	Lump-L / Dist-D f_o(GHz)	Input Limitation due to: L Flat	C Slope	L ½	C Slope	L Full	C Slope	Output Limitation due to: L Flat	C Slope	L ½	C Slope	L Full	C Slope
(a)	6.10	4-8	L	.93	.93	1	1	1	1	.994	1	1	1	1	1
			$D(f_o= 12)$	.897	.915	1	1	1	1	.963	1	1	1	1	1
			$D(f_o= 16)$	.927	.913	1	1	1	1	.981	1	1	1	1	1
(b)	5.88	8-12	L	-	1	-	1	-	1	.984	-	1	-	1	-
			$D(f_o= 18)$	-	1	-	1	-	1	.827	-	.993	-	1	-
			$D(f_o= 24)$	-	1	-	1	-	1	.914	-	1	-	1	-
(c)	5.99	1-25	L	.984	-	1	-	1	-	1	1	1	1	1	1
			$D(f_o= 3.75)$	.972	-	1	-	1	-	.994	.997	1	1	1	1
			$D(f_o= 5$	.972	-	1	-	1	-	.998	.999	1	1	1	1
(d)	5.74	8-12	L	-	.997	-	1	-	1	1	-	1	-	1	-
			$D(f_o= 18)$	-	.852	-	1	-	1	.986	-	1	-	1	-
			$D(f_o= 24)$	-	.950	-	1	-	1	.997	-	1	-	1	-

NOTE: A '1' entry implies gain greater than .9990

suggest that the conventional approach of designing one matching network for flat response and the other for full slope compensation may not yield optimal results in this context, and that a division of the slope-compensation role between the two networks may offer advantages. Table II gives limitations for the devices under study. It is also noted, as expected, that performance is better for the lumped-element cases than for those using commensurate-distributed networks and in this case better performance is obtained for the higher f_0. These are, however, merely indications as such calculations are subject to inaccuracies inherent in device modeling.

III. Matching Network Design

The next step in the design procedure is concerned with the design of matching networks capable of absorbing the model elements, having suitable slope characteristics, and capable of realizing any impedance transformation necessary. Techniques discussed are a) the analytical approach following the technique of Levy [8] and others, b) Remez algorithm using lumped elements, and c) Remez algorithm using commensurate-distributed elements. All examples use the devices given, with the bandwidth in each case being that for which data are given in Table I.

A. "Analytical" Approach for the Design of Flat Bandpass Networks

It is initially necessary to transform model element restrictions to restrictions on an equivalent low-pass structure. In the case of a single constraint, the Levy procedure permits selection of k and ϵ in the relation

$$G_m = \frac{1}{1+k^2+\epsilon^2 T_n^2(x)} \tag{3}$$

with G_m being matching network gain and where T_n is the nth order Chebyshev polynomial and x is normalized frequency. In the case of two constraints as in the RLC output model case, it may be that the g_2 of the procedure (prototype normalized value, [8]) may be adequate for absorption; but if not, a solution as in [9] may be obtained. The matching network may be derived from the prototype using low-pass to bandpass techniques and impedance transformation using the Shea algorithm ([8, fig. 6]) may be implemented.

This design procedure was used to design flat-response matching networks for the devices over the bandwidth as considered earlier. The minimum-degree flat-response networks consistent with impedance transformation being possible are shown in Figs. 3 and 4. Results will be discussed later in the light of other approaches.

B. Remez Algorithm—Lumped Case

As noted, the approach in Section III-A has been restricted to flat-response lumped-element cases with equal numbers of zeros at $\omega=0$ and $\omega=\infty$. A procedure of wider applicability is that of using the Remez algorithm approach of Ku *et al.* [10].

Restricting transmission zeros to lie at $\omega=0$ or $\omega=\infty$, the matching network response is of the form

$$G_m(x) = \frac{Kx^m}{P_n(x)}, \qquad \text{where } x=\omega^2. \tag{4}$$

By an iterative technique, it is possible to calculate the points within the band of interest where the departure from the ideal characteristic is maximum. This departure

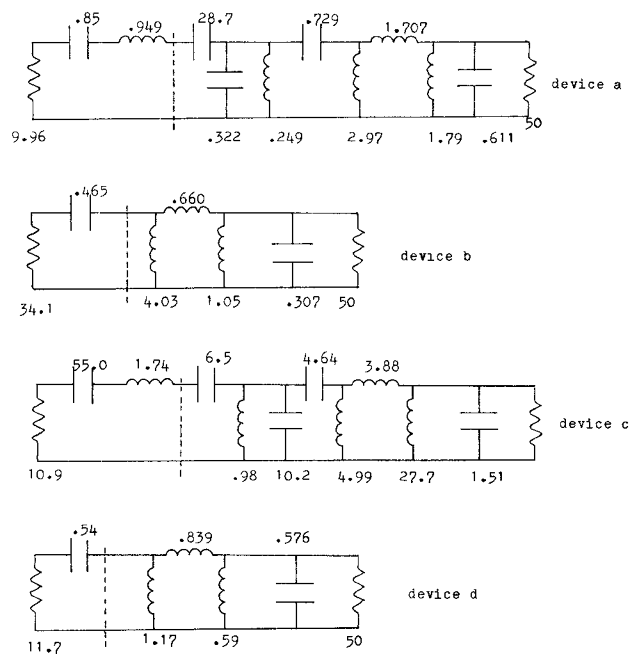


Fig. 3. Lumped-element matching networks (of minimum degree consistent with impedance-transformation requirements) for input, designed using "analytic" approach of Section III-A. Capacitance values are in picofarads, inductance values in nanohenries, and resistance in ohms (in this and all other figures unless stated).

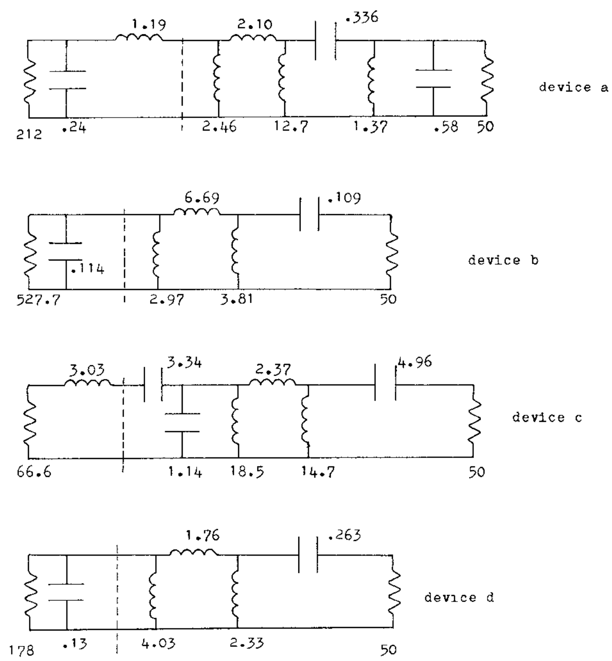


Fig. 4. Lumped-element matching networks (of minimum degree consistent with impedance transformation requirements) for output, designed using "analytic" approach of Section III-A.

may be termed δ where

$$\delta = \ln G_m(x) - \ln f(x) \tag{5}$$

and $f(x)$ is the desired uniform-slope response. The coefficients of the polynomial $P_n(x)$ are thus easily calculated. The scaling factor K is chosen so that $G_m(x) \leq 1$ for all real x, where $G_m(x)$ is the matching network gain.

Having obtained such a transmission function, network synthesis is possible using the usual techniques. With arbi-

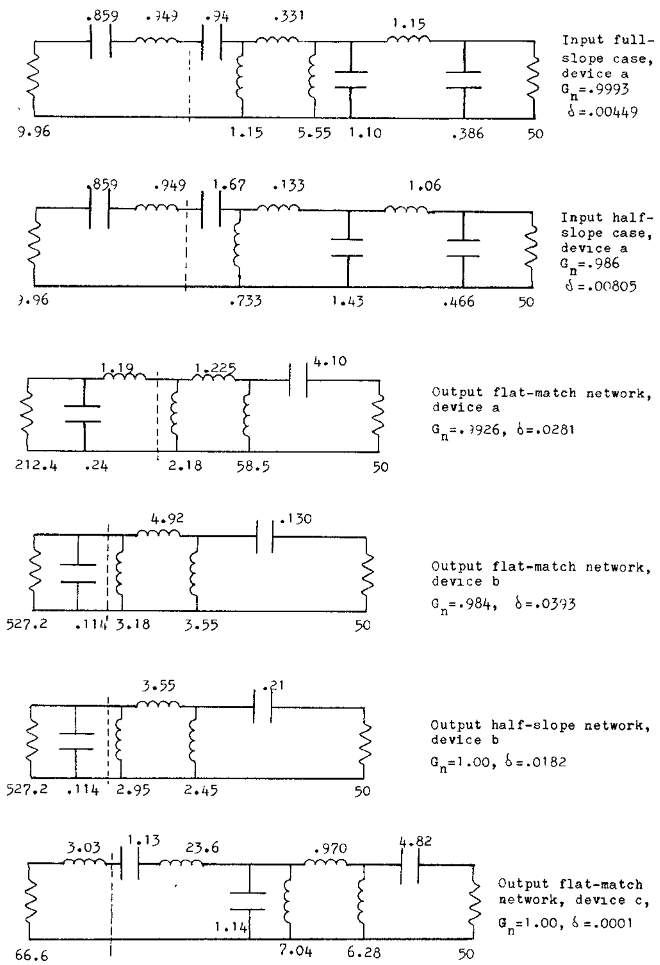


Fig. 5. Networks using lumped elements designed using approach of Section III-B. The simpler networks have 2 transmission zeros at zero and 2 at infinity; the more complicated structures have 2 transmission zeros at infinity and 4 at zero frequency.

trary selection of the K and δ terms, there is no guarantee that element absorption is achieved. To achieve optimal performance, an objective function was defined as

$$U=(1-G_n)+\delta+\text{penalty terms} \tag{6}$$

with G_n being maximum gain and penalty terms given by

$$P=10^4\left|\frac{y-y_d}{y_d}\right|^2 \tag{7}$$

being introduced as required, where y_d is the desired or limiting value of the element y. Numerical optimization was then used to select optimum δ and G_n. Some results of network design using this approach are shown in Fig. 5 and a comparative discussion appears in Section III-D.

C. Remez Algorithm—Distributed Case

In the commensurate-distributed case, one must consider the use of unit elements, implying that the gain response of the network with real-frequency zeros of transmission (in the transformed variable) at 0 and is of the form [9], (where $x=\Omega^2$, Ω being the transformed real-frequency variable)

$$G_m(x)=\frac{x^m(1+x)^q}{P_n(x)} \tag{8}$$

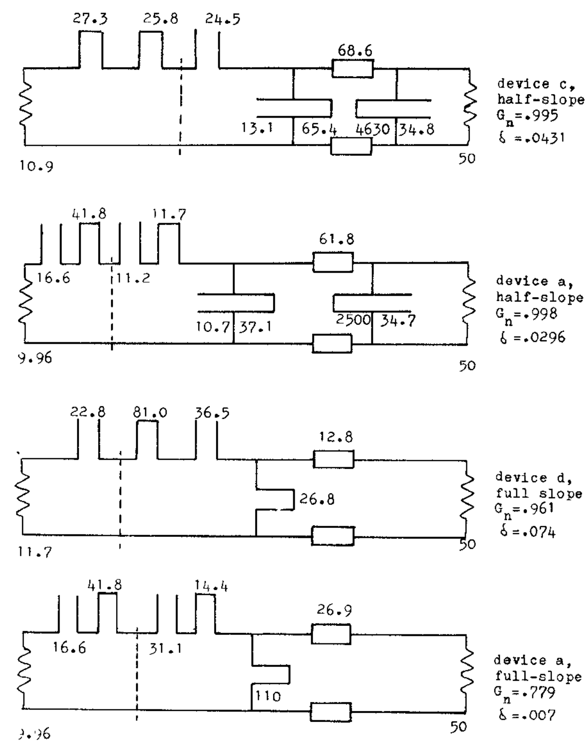


Fig. 6. Input-matching networks using commensurate-distributed elements. Figures given are for characteristic impedance in ohms. All lines are of commensurate length, being of free space value 0.469 cm (device a), 0.313 cm (devices b and d), and 1.5 cm (device c).

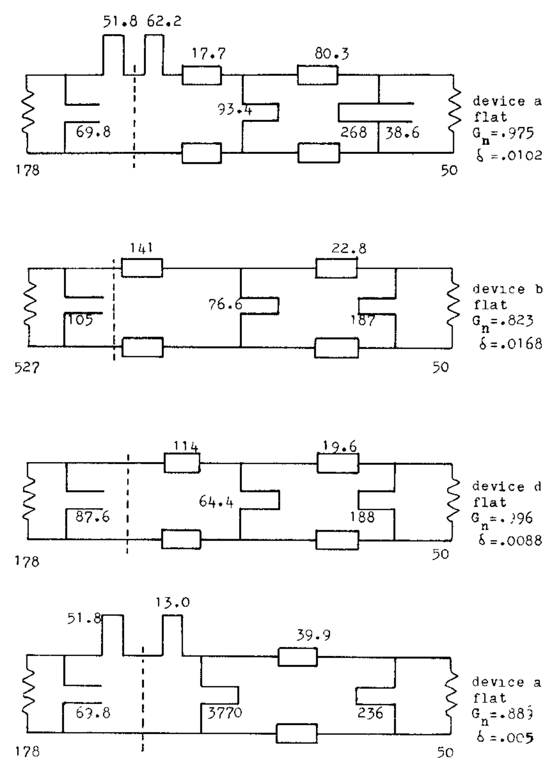


Fig. 7. Output matching networks using commensurate-distributed elements (lengths as in Fig. 6).

with m high-pass elements, q unit elements, and $n-(m+q)$ low-pass elements.

Approximation proceeds as before but the synthesis

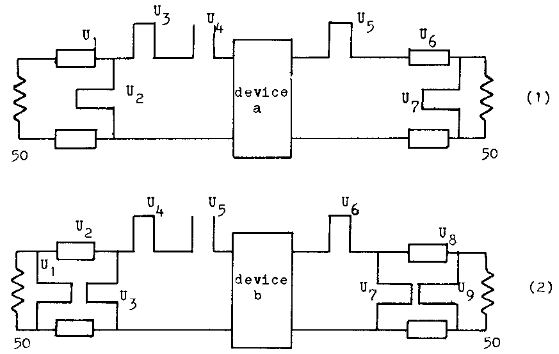


Fig. 8. Amplifiers with commensurate-distributed matching networks. Element values before and after optimization are given in Table III.

procedure differs. It is necessary to use the Richards extraction procedure for each unit element and impedance transformation using an approach such as the Shea or Norton transformation is not possible due to restrictions on contiguous stub placement. The Carlin–Kohler technique using partial extraction of a shunt inductive stub was used. [11] The procedure was set up for optimization of G_n and δ with the objective function of (6). Examples of designs using this approach are shown in Figs. 6 and 7. In general, gain, and ripple quantities are found to be inferior to those obtained in the case of lumped networks.

D. Discussion of the Matching Network Design Techniques

In evaluating the suitability of a design technique for matching networks it is necessary to consider ease of absorption of model elements and the provision of impedance transformation to the desired termination resistance. Gain–bandwidth calculations provide an indication as to capability to absorb model elements for a particular gain and slope combination. Such considerations do not, however, give indications as to the feasibility of impedance transformation capability, avoiding the requirement for an ideal transformer.

It is noted that a strict application of optimal gain–bandwidth design approaches such as considered in Section III-A may lead to problems in accomodating this impedance transformation requirement, whereas the Remez algorithm design approaches with optimization of G_n and δ considering impedance transformation in the penalty function terms may lead to a simpler network with minimal loss of performance in the gain–bandwidth sense. For example in the case of device a, the strict gain–bandwidth design required a fourth-order network while a numerically designed network can yield a simpler network giving satisfactory response.

The distributed matching networks obtained in the case of the devices under study indicate that impedance transformation is often a constraint, as indicated by the design procedure yielding very high values of shunt inductance (in practice omitted) at either point in the partial extraction procedure. Once again, it is difficult to predict achievable ranges of impedance transformation with a particular topology for given bandwidth and slope. It is noted that gain–ripple properties seem somewhat inferior to those achievable using lumped elements.

TABLE III
ELEMENT VALUES (a) PRIOR TO AND (b) POST OPTIMIZATION FOR FIG. 8

	(1)		(2)	
	a	b	a	b
U_1(Z) Ω	26.[illegible]	16.[illegible]	115	138
U_1(l) mm	4.1	3.9	3.1	3.3
U_2(Z)	110	178.5	25.7	26.1
U_2(l)	4.1	2.1	3.1	4.3
U_3(Z)	14.4	11.2	179	209
U_3(l)	4.1	4.7	3.1	3.9
U_4(Z)	31.1	13.[illegible]	37.0	113
U_4(l)	4.1	4.1	3.1	2.6
U_5(Z)	13.0	8.7	67.8	35.4
U_5(l)	4.1	2.7	3.1	4.8
U_6(Z)	32.9	28.1	112	173
U_6(l)	4.1	0.5	3.1	3.1
U_7(Z)	236	135	27.7	19.2
U_7(l)	4.1	3.7	3.1	4.1
U_8(Z)			131	126
U_8(l)			3.1	3.0
U_9(Z)			173	84.4
U_9(l)			3.1	2.1

IV. OVERALL AMPLIFIER PERFORMANCE CONSIDERATIONS

In terms of overall amplifier performance, one is interested in gain response, perhaps in noise figure, perhaps in VSWR, and in stability if the device is not unconditionally stable. The matching networks used may be lossless or dissipative (e.g., [6]).

In this discussion, gain performance is considered to be of primary interest and lossless networks designed according to the procedures of Section III were used at both ports. Optimization procedures following [12] were incorporated in a network analysis and optimization program termed ACADAMP, with an objective function U_i at frequency f_i given by

$$U_i = (G_t - 0.9G_m(f_H))^2 \tag{9}$$

where G_t is transducer power gain at frequency f_i and $G_{cm}(f_H)$ is the conjugate matched gain at the highest frequency in the band.

Some results using the transistors with matching networks before and after optimization of element values are shown (Figs. 8–11). It is seen that useful initial designs are obtainable using these techniques and that optimization

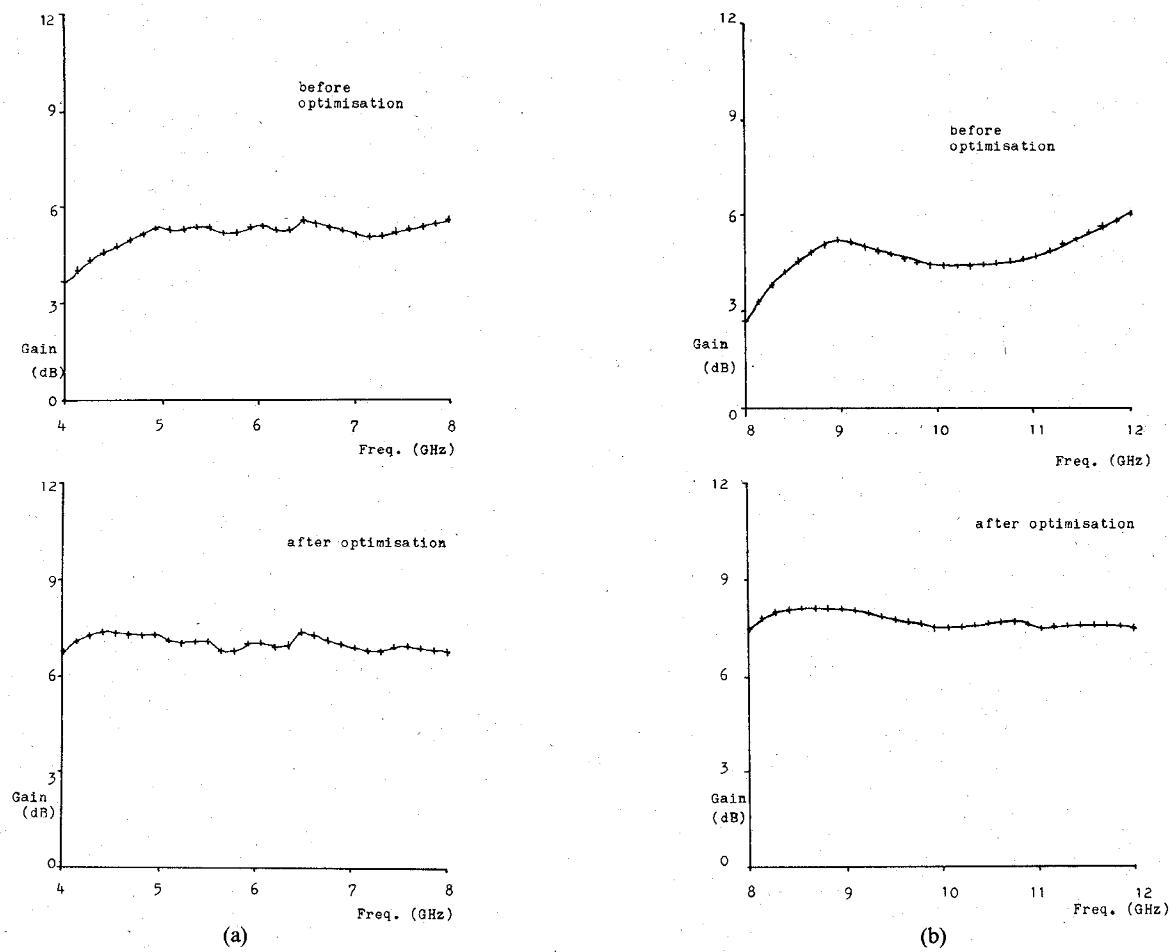


Fig. 9. (a) Gain response of network (1) of Fig. 8 before and after optimization. (b) Gain response of network (2) of Fig. 8 before and after optimization.

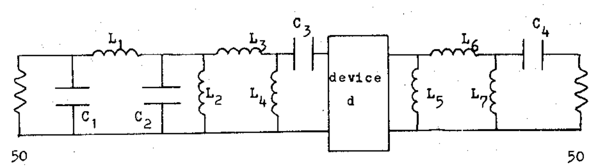
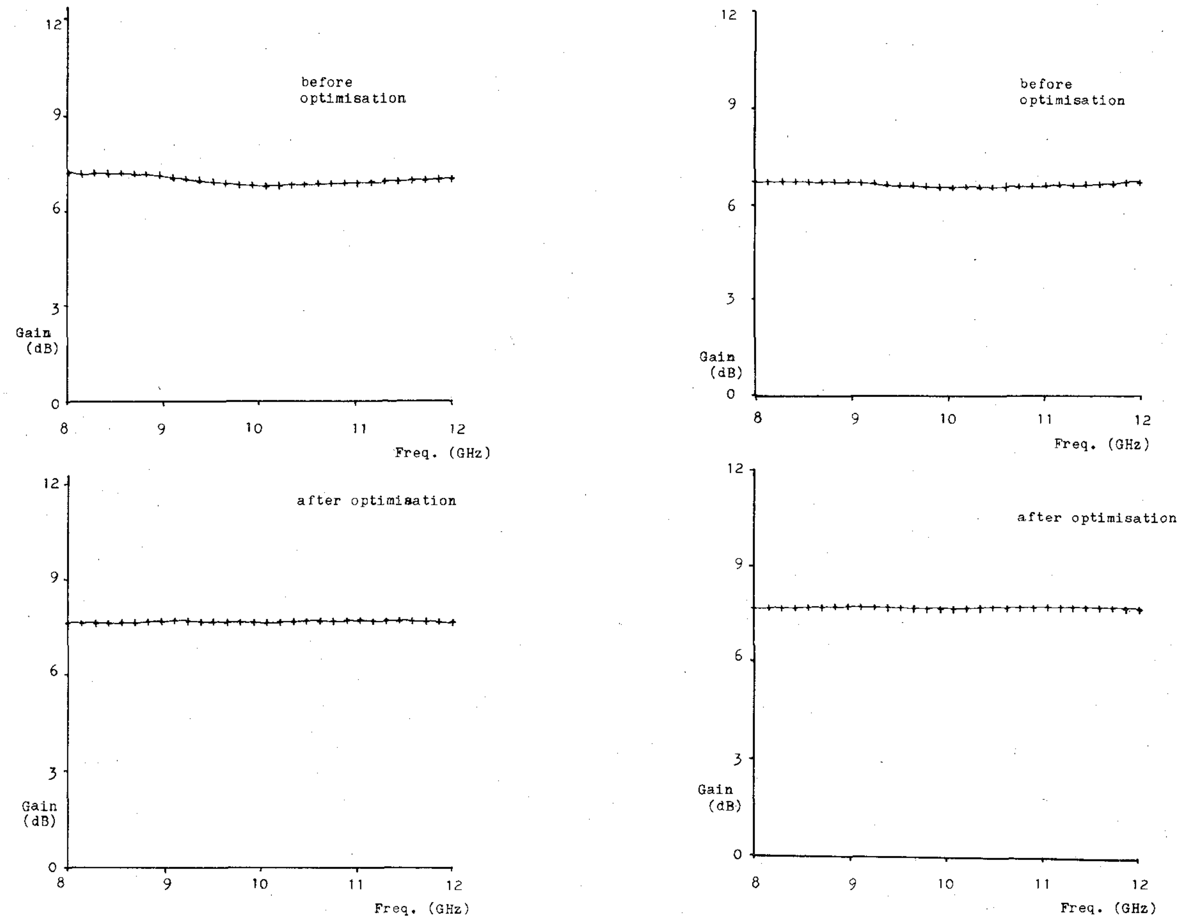


Fig. 10. Networks with device d, (1) full-slope input, flat output match and (2) equally divided slope-compensation case. Element values before and after optimization are given in Table IV.

TABLE IV
ELEMENT VALUES FOR FIG. 10 FOR (1) "FULL/FLAT" SLOPES CASE AND (2) FOR CASE OF EQUALLY DIVIDED SLOPE IN EACH MATCHING NETWORK

	(1)		(2)	
	a	b	a	b
C_1 (pF)	.184	.260	.197	.300
L_1 (nH)	.709	.681	.694	.608
C_2 (pF)	.546	.634	.597	1.00
L_2 (nH)	1.85	1.58	1.52	.674
L_3 (nH)	.989	1.03	.761	.698
L_4 (nH)	2.61	2.05	1.88	1.55
L_5 (nH)	.366	3.71	2.36	4.37
C_3 (pF)	.554	.788	1.77	2.86
L_6 (nH)	1.95	1.54	.689	.698
L_7 (nH)	2,60	3.38	1.90	1.72
C_4 (pF)	.237	.225	1.87	1.05

(*a* values are for initial design; *b* values are after optimization)



(a) (b)

Fig. 11. (a) Gain response of network (1) of Fig. 10 before and after optimization. (b) Gain response of network (2) of Fig. 10 before and after optimization.

can yield suitable responses. Computation of VSWR figures indicates, as expected, that VSWR at low frequencies is very high in the case of full-slope networks, as is expected due to use of lossless elements. Similarly, a flat-response network gives a low VSWR, with less extreme values obtainable using a "divided-slope" approach.

V. Conclusions

In this paper, design techniques using device modeling and network synthesis procedures, augmented by numerical methods, have been discussed. This approach may be viewed as complementary to that of Carlin [13], and it will usually give acceptable gain performance after suitable optimization. While this procedure is not designed to offer guaranteed stability in the case of a potentially unstable device, no stability problems were encountered in the case of the potentially unstable device studied.

Gain–bandwidth restrictions suggest that lumped-element networks, and those design approaches apportioning the slope-compensation between input and output matching networks, may offer optimal performance. Impedance transformation capability is seen to be a major constraint, particularly in the case of commensurate-distributed structures. The use of the Remez algorithm approach [10], with the extension proposed in this paper, where gain and ripple quantities are adjusted to facilitate exact absorption and impedance transformation, is a particularly useful approach, although somewhat demanding in computer time due to repeated synthesis operations required in the optimization procedure. It is, of course, necessary to choose a structure with appropriate numbers of transmission zeros (real frequency) at 0 and ∞, and any fully automated design procedure would require evaluation of alternative options.

The important role of numerical optimization procedures is evident. Many errors are, of course, introduced in the simplifications necessary for use of the design procedures discussed, and overall trimming of component values using numerical optimization is usually required.

References

[1] W. H. Ku, M. E. Mokari-Bolhassan, W. C. Petersen, A. F. Podell, and R. R. Kendall, "Microwave octave-band GaAs FET amplifiers," in *Proc. 1975 Int. Microwave Symp.*, (Palo Alto, CA), pp. 69–72.

[2] N. A. Slaymaker, R. A. Soares, and J. A. Turner, "GaAs MESFET small-signal X-band amplifiers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 329–337, June 1976.

[3] W. H. Ku and A. F. Podell, "New results in the design of microwave bipolar and FET amplifiers," in *Proc. 1974 IEEE MTT Int. Symp.*, pp. 357–359.

[4] W. H. Ku and W. C. Petersen, in *Proc. IEEE Int. Symp. on Circuits and Systems*, pp. 704–707, 1977.

[5] S. O. Scanlan and G. P. Young, "Error considerations in the design of microwave transistor amplifiers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 1163–1169, Nov. 1980.

[6] C. A. Liechti and R. L. Tillman, "Design and performance of microwave amplifiers with GaAs Schottky field-effect transistors," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 510–517, May 1974.
[7] G. E. Bodway, "Two-port power flow analysis using generalised scattering parameters," *Microwave J.*, vol. 10, May 1967.
[8] R. Levy, "Explicit formulas for Chebyshev impedance-matching networks, filters and interstages," *Proc. Inst. Elec. Eng.*, vol. 111, pp. 1099–1106, June 1964.
[9] J. O. Scanlan and J. T. Lim, "The effect of parasitic elements on reflection type tunnel diode amplifier performance," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 827–836, Nov. 1965.
[10] W. H. Ku, W. C. Petersen, and A. F. Podell, "Synthesis of a class of broadband matching networks for transistor amplifiers," in *Proc. 9th Asilomar Conf. on Circuits, Systems and Computers* (Nov. 1975), pp. 211–216.
[11] H. J. Carlin and W. Kohler, "Direct synthesis of bandpass transmission-line structures," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 282–297, May 1965.
[12] K. Madsen and H. Schjaer-Jacobsen, "Fortran subroutines for nonlinear minimax optimisation," Rep. 76-15, Numerisk Institut, DK-2800 Lyngby, Denmark, 1976.
[13] H. J. Carlin, "A new approach to gain-bandwidth problems," *IEEE Trans. Circuits Syst.*, vol. CAS-24, pp. 170–175, Apr. 1977.

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Coupled Slots on an Anisotropic Sapphire Substrate

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Abstract—**Two analytical approaches are presented for coupled slots on an anisotropic sapphire substrate using the network analytical methods of electromagnetic fields. One is based on the quasi-static approximation and it derives the transformation from the case with the anisotropic substrate to the case with the isotropic substrate. The other is based on the hybrid mode formulation and it gives the dispersion characteristics.**

I. Introduction

PROPAGATION CHARACTERISTICS of coplanar transmission lines have received considerable attention. The slot line [1] has been analyzed using the hybrid mode formulation [2]–[4]. However, the dispersion characteristic of coupled slots has been evaluated based on the first-order approximation [3], [5]. Moreover, only the case with isotropic substrate has been treated.

In this paper, a method of analysis of coupled slots on an isotropic and/or anisotropic substrate is presented. This method contains two approaches: the quasi-static and hybrid mode formulation. The quasi-static approach, based on the network analytical methods of electromagnetic fields [6] and variational techniques, shows that coupled slots on an anisotropic sapphire substrate can be transformed into the equivalent coupled slots on an isotropic substrate. The hybrid mode formulation, which is an extension of the treatment in [5], gives the dispersion characteristics of the dominant and first higher order mode, and it suggests the close relation between the first higher order mode of coupled slots and the TM_0 surface wave of a sapphire-coated conductor.

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II. Electromagnetic Field Representations

The cross section of the coupled slots on a single-crystal sapphire substrate is shown in Fig. 1. The sapphire crystal is uniaxial, but is anisotropic. The permittivity tensor of sapphire is the diagonal matrix as follows:

$$\hat{\epsilon}=\epsilon_0\begin{bmatrix}\epsilon_\perp & 0 & 0\\ 0 & \epsilon_\perp & 0\\ 0 & 0 & \epsilon_{11}\end{bmatrix}. \quad (1)$$

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